Motion in a straight line

Note : Answers given below are intended only as a reference for the questions asked in board examinations. These answers are NOT an attempt at a detailed discussion related to the question.

VSAQs

1. The states of motion and rest are relative. Explain

State of a body, as observed by an observer, depends on the state of the observer also.

Example : Consider two persons traveling in a bus moving at velocity v w.r.t. the ground. Relative velocity of A w.r.t. B or B w.r.t. A is zero. For an observer on the ground both A and B are observed to be moving with velocity v.

2. How is average velocity different from instantaneous velocity?

Average velocity gives an overall estimate of motion of the body. Average velocity is obtained by considering the total displacement of the body in a given interval of time i.e. $\bar{v}_{\text{avg}} = \dfrac{\bar{S}_{\text{total}}}{t_{\text{total}}}$

Instantaneous velocity gives the state of motion of the body at a particular instant of time. It is obtained by considering the displacement of the body in a very small interval of time i.e. $\bar{v}_{\text{inst}} = \dfrac{d\bar{S}}{dt}$

3. Given an example where velocity of a body is zero but its acceleration is not zero.

At the highest point of the trajectory of a vertically projected body, the velocity is momentarily zero where as its acceleration (due to gravity) is not zero

4. A vehicle travels half the distance L with speed v_1 and the other half with speed v_2. What is the average speed?

Average speed is given by

$$\bar{v}_{\text{avg}} = \frac{\bar{S}_{\text{total}}}{t_{\text{total}}}$$

Total distance is $2L$

$$t_1 = \frac{L}{v_1} \quad \text{and} \quad t_2 = \frac{L}{v_2}$$

Using these in eq (i)

$$v_{\text{avg}} = \frac{2L}{\dfrac{L}{v_1} + \dfrac{L}{v_2}}$$

$$v_{\text{avg}} = \frac{2v_1v_2}{v_1 + v_2}$$

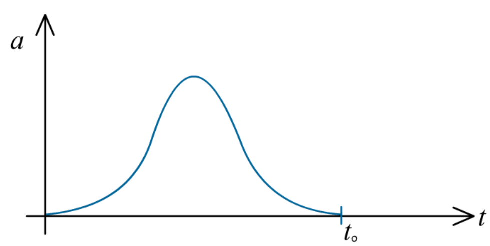
5. A lift coming down is just about to reach the ground floor. Taking the ground floor as origin and positive direction upwards for all quantities, which one of the following is correct?

a) $x < 0, v < 0, a > 0$
b) $x > 0, v < 0, a < 0$
c) $x > 0, v < 0, a > 0$
d) $x > 0, v > 0, a > 0$

The body is above the ground hence its position coordinate $x > 0$, velocity is downwards therefore $v < 0$ and acceleration is upwards (because the lift is slowing down) therefore $a > 0$.

6. A uniformly moving cricket ball is hit with a bat for a very short time and is turned back. Show the variation of its acceleration with time taking the acceleration in the backward direction as positive.

Assume that the ball comes in contact with the bat at $t = 0$ and remains in contact for a small interval of time t_o. During this interval of time acceleration of the ball is in the left hand side direction.



7. Given an example of a one dimensional motion where a particle moving along the positive x-direction comes to rest periodically and moves forward.

Consider a spring mass system mounted on a platform moving horizontally with uniform velocity. Then net motion of body due to its oscillations on the platform and the motion of the platform is such that it comes to rest momentarily and then moves along x -direction.

8. An object falling through fluid is observed to have an acceleration given by $a = g - bv$ where g is gravitational acceleration and b is a constant. After a long time is observed to fall with constant velocity. What would be the value of this constant velocity?

Given $a = g - bv$
When acceleration becomes zero we get

$0 = g - bv$
$bv = g$
$v = g/b$

9. If the trajectory of a body is parabolic in one frame, can it be parabolic in another frame that moves with a constant velocity with respect to the first frame? If not, what can it be?

Yes the path of projectile may be parabolic in another frame of reference which is moving with constant velocity (v_F) such that any one of the components of motion of the projectile is not equal to the velocity of frame.

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10. A spring with one end attached to a mass and the other to a rigid support is stretched and released. When is the magnitude of acceleration the maximum?

Magnitude of acceleration is the maximum when the spring is in a state of maximum compression or extension. This is because force exerted by the spring is given by $F = -kx$.

SAQs

1. Can the equations of kinematics be used when acceleration varies with time? If not, what form would these equations take?

No, the following equations of kinematics <u>cannot</u> be used when acceleration of the body is not constant

$$v = u + at, \quad S = ut + \frac{1}{2}at^2 \quad \text{and } v^2 - u^2 = 2aS$$

The general equations of motion that can be used for both constant and variable accelerations are

$$\bar{v} = \frac{d\bar{S}}{dt} \quad \Rightarrow \quad S = \int v \, dt$$

$$\bar{a} = \frac{d\bar{v}}{dt} \quad \Rightarrow \quad \bar{v} = \int a \, dt$$

2. A particle moves in a straight line with uniform acceleration. Its velocity at $t = 0$ is v_1 and at $t = t$ is v_2. Average velocity of the particle in this interval is $(v_1 + v_2)/2$. Is this correct? Substantiate your answer.

Yes the given relation is correct.

Average velocity is given by $\bar{v}_{avg} = \dfrac{\bar{S}_{total}}{t_{total}}$ ----(I)

As the motion is with constant acceleration, the total displacement is given by the relation

$$S = v_1 t + \frac{1}{2}at^2 \quad \text{--- (II)}$$

Using $v = u + at$ we get $v_2 = v_1 + at$

$$a = \frac{v_2 - v_1}{t} \quad \text{--- (III)}$$

Substituting this in eq (II)

$$S = v_1 t + \frac{1}{2}\left[\frac{v_2 - v_1}{t}\right]t^2$$

$$S = \left[\frac{v_1 + v_2}{2}\right]t$$

Using the relation for $S = \frac{t}{a}$ average

$$v_{\text{avg}} = \frac{1}{t}\left[\frac{v_1 + v_2}{2}\right]t$$

$$v_{\text{avg}} = \left[\frac{v_1 + v_2}{2}\right]$$

3. Can velocity of an object be in a direction other than the direction of acceleration of the object? If so, give an example.

Yes, velocity of a body can be in a direction other than the direction of acceleration

Examples

(a) When a car decelerates its velocity and acceleration are in opposite direction

(b) During the ascent of a vertically projected body its acceleration is in the downward direction while it velocity is in the upward direction

(c) For a body in uniform circular motion, instantaneous velocity (which is tangential) is perpendicular to the instantaneous acceleration (which is radially inwards)

4. A parachutist flying in an aeroplane jumps when it is at a height of 3km above the ground. He opens his parachute when he is about 1km above the ground. Describe his motion.

Considering only the vertical component of his velocity, his initial vertical velocity may be assumed to be zero.

Under the influence of acceleration due to gravity, he gains velocity in the downward direction. This increase in velocity leads to an increase in viscous force due to air which tends to oppose his motion. As a result of these two forces, he attains a terminal velocity.

Once he opens the parachute, the upwards force due to air increases and he undergoes further deceleration. He reaches the ground with this new lower terminal velocity which helps him land safely.

5. A bird holds a fruit in its beak and flies parallel to the ground. It lets go off the fruit at some height. Describe the trajectory of the fruit as it falls to the ground as seen by (a) the bird (b) a person on the ground.

(a) for the bird, the seed appears to fall down vertically. This is because horizontal velocity of the bird is equal to the horizontal component of the seed. The vertical component of the falling seed causes its vertical displacement w.r.t. the bird

(b) For a person on ground the seed appears to follow a parabolic path. This is because initial velocity (in the horizontal direction) causes the horizontal movement of the seed and acceleration due to gravity causes the seed to descend.

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6. A man runs across the roof of a tall building and jumps horizontally on to the (lower) roof of an adjacent building. If his speed is 9 ms^{-1}, horizontal distance between the buildings is 10m and height difference between the roofs is 9m, will he be able to land on the next building? (take g = 10 ms^{-2})

Initial velocity (in horizontal direction) : 9ms^{-1}
Initial velocity (in vertical direction) = 0
Difference in heights : 9 m

$TOD = \sqrt{\frac{2H}{g}} = 3s$

Horizontal distance covered in 3 s is given by
$R = u_x \times TOD = 9 \times 3 = 27m$
As this is greater than the horizontal separation between the buildings, the person will be able to reach the building

7. A ball is dropped from the roof of a tall building and simultaneously another ball is thrown horizontally with some velocity from the same roof. Which ball lands first? Explain your answer.

Both the balls will reach the ground simultaneously.
Let H be the height from which both the bodies are dropped.
The ball that is dropped freely has zero initial velocity in the vertical direction. Its acceleration and displacement in the vertical direction are $-g$ and $-H$ respectively. Time of descent of the body is given by

$TOD_1 = \sqrt{\frac{2H}{g}}$

The ball that is projected horizontally also has zero initial velocity in the vertical direction. Its acceleration and displacement in the vertical direction are also $-g$ and $-H$ respectively. Therefore time of descent of the second body is also given by

$TOD_2 = \sqrt{\frac{2H}{g}}$

Since $TOD_1 = TOD_2$ both the bodies reach the ground simultaneously

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8. A ball is dropped from a building and simultaneously another ball is projected upward with some velocity Describe the change in relative velocities of the balls as a function of time.

Initial velocity of the ball that is dropped is zero therefore

$v_A = 0 + (-g)t$

$\bar{v}_A = -gt\ \hat{j}$

Let the initial velocity of the vertically projected ball be u. Therefore

$$v_B = u + (-g)t$$

$$\bar{v}_B = [u - gt]\hat{j}$$

Relative velocity of A w.r.t. B is

$$\bar{v}_{AB} = \bar{v}_A - \bar{v}_B$$

$$\bar{v}_{AB} = \left(-gt\ \hat{j}\right) - (u - gt)\hat{j}$$

$$\bar{v}_{AB} = -u\hat{j}$$

Therefore it is observed that relative velocity of A w.r.t. B is constant and its magnitude is equal to the initial velocity of projection of B

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9. A typical rain drop is about 4mm is diameter. If a raindrop falls from a cloud which is at 1km above the ground, estimate its momentum when it hits the ground.

Diameter of the rain drop = 4mm

Radius of the rain drop = 2mm

Mass of the raindrop = density x $\frac{4}{3}\pi r^3$

$m = 10^3 \times \frac{4}{3}\pi(2\times10^{-3})^3 = 3.35 \times 10^{-5}$ kg

$v = \sqrt{2gh} = 140\ \text{ms}^{-1}$

$p = mv$

$p = 0.0047\ \text{kgms}^{-1}$

10. Show that the maximum height reached by a projectile at an angle of 45° is one quarter of its range.

Range is given by $R = \dfrac{u^2\sin(2\theta)}{g}$ and height is given by $H = \dfrac{u^2\sin^2(\theta)}{2g}$

Given angle of projection is $\theta = 45°$ therefore

$$R = \frac{u^2\sin(2\times45)}{g} \quad \text{and} \quad H = \frac{u^2\sin^2(45)}{2g}$$

$$R = \frac{u^2}{g} \text{ and } \quad H = \frac{u^2}{2g}\times\frac{1}{2}$$

Therefore $H = R/4$